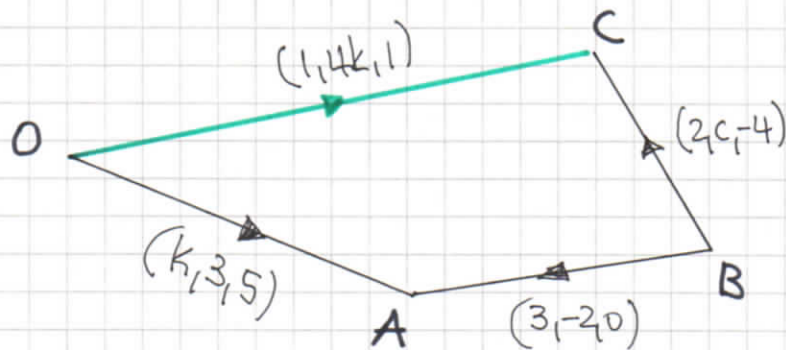


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## 1YGB-MP2 PAPER 2 - QUESTION 1

STARTING WITH A VECTOR DIAGRAM



$$\begin{aligned} A & (k, 3, 5) \\ \vec{BA} & = (3, -2, 0) \\ \vec{BC} & = (2, c, -4) \\ C & (1, 4k, 1) \end{aligned}$$

FORMING A VECTOR EQUATION

$$\Rightarrow \vec{OA} + \vec{AB} + \vec{BC} = \vec{OC}$$

$$\Rightarrow (k, 3, 5) - (3, -2, 0) + (2, c, -4) = (1, 4k, 1)$$

$$\Rightarrow (k-1, c+5, 1) = (1, 4k, 1)$$

$$[i]: k-1=1 \Rightarrow \underline{k=2}$$

$$[j]: c+5=4k$$

$$c+5=8$$

$$\underline{c=3}$$

FINALLY WE CAN FIND THE DISTANCE BC

$$\Rightarrow |\vec{BC}| = |2, 3, -4|$$

$$\Rightarrow |\vec{BC}| = \sqrt{2^2 + 3^2 + (-4)^2}$$

$$\Rightarrow |\vec{BC}| = \sqrt{4+9+16}$$

$$\Rightarrow |\vec{BC}| = \underline{\underline{\sqrt{29} \approx 5.39}}$$

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## LYGB - MP2 PAPER 2 - QUESTION 2

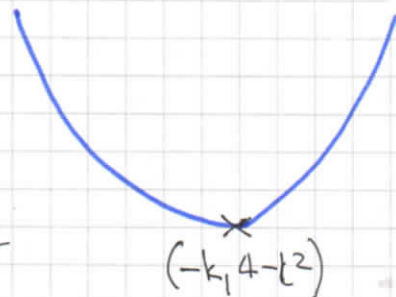
a) COMPLETING THE SQUARE

$$f(x) = x^2 + 2kx + 4, \quad x \in \mathbb{R}$$

$$f(x) = (x+k)^2 - k^2 + 4$$

$f(x)$  HAS A MINIMUM VALUE OF  $4 - k^2$

$$\underline{f(x) \geq 4 - k^2} //$$



b)  $f(g(2)) = 4$

$$\Rightarrow f(3 - k \times 2) = 4$$

$$\Rightarrow f(3 - 2k) = 4$$

$$\Rightarrow (3 - 2k)^2 + 2k(3 - 2k) + 4 = 4$$

$$\Rightarrow 9 - 12k + 4k^2 + 6k - 4k^2 = 0$$

$$\Rightarrow 9 = 6k$$

$$\Rightarrow \underline{k = \frac{3}{2}} //$$

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# YOB - MP2 PAPER R - QUESTIONS

START FORMING EQUATIONS AS FOLLOWS

$$\begin{array}{ccc} u_2 & & u_3 & & u_9 \\ \alpha+d & & \alpha+2d & & \alpha+8d \end{array}$$

$\xrightarrow{\times r}$                        $\xrightarrow{\times r}$

←  $u_n = a + (n-1)d$

$$\Rightarrow \begin{cases} (\alpha+d)r = \alpha+2d \\ (\alpha+2d)r = \alpha+8d \end{cases}$$

ELIMINATE THE COMMON RATIO  $r$ , BY DIVISION

$$\Rightarrow \frac{\alpha+d}{\alpha+2d} = \frac{\alpha+2d}{\alpha+8d}$$

$$\Rightarrow (\alpha+d)(\alpha+8d) = (\alpha+2d)^2$$

$$\Rightarrow \cancel{\alpha^2} + 8\alpha d + \alpha d + 8d^2 = \cancel{\alpha^2} + 4\alpha d + 4d^2$$

$$\Rightarrow 4d^2 + 5\alpha d = 0$$

$$\Rightarrow d(4d + 5\alpha) = 0$$

$$\Rightarrow 5\alpha + 4d = 0 \quad (d \neq 0)$$

$$\Rightarrow d = -\frac{1}{5}\alpha$$

NOW RETURNING & PICKING ONE OF THE ORIGINAL EQUATIONS WHICH CONTAIN  $a, d$  &  $r$

$$\Rightarrow (\alpha+d)r = \alpha+2d$$

$$\Rightarrow \left(\alpha - \frac{1}{5}\alpha\right)r = \alpha + 2\left(-\frac{1}{5}\alpha\right)$$

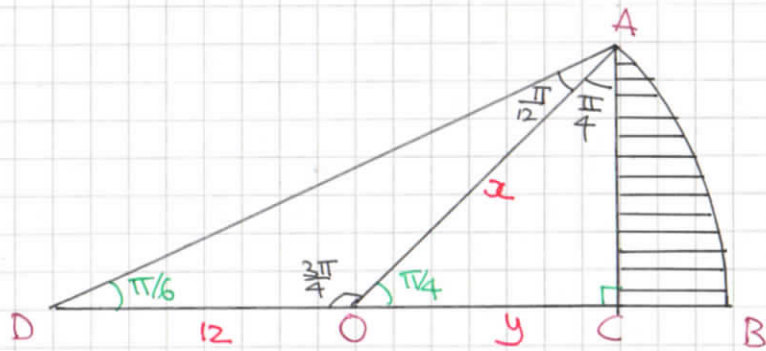
$$\Rightarrow \frac{4}{5}\alpha r = \frac{3}{5}\alpha$$

$$\Rightarrow 4\alpha r = 3\alpha$$

$$\Rightarrow r = \frac{3}{4}$$

# LYGB - MP2 PAPER R - QUESTION 4

a) STARTING WITH A DIAGRAM



OBTAIN SOME ANGLES

- $\widehat{DOA} = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$  (straight line)
- $\widehat{DAO} = \pi - \left(\frac{\pi}{6} + \frac{3\pi}{4}\right) = \frac{\pi}{12}$  (triangle  $\triangle DAO$ )
- $\widehat{OAC} = \pi - \left(\frac{\pi}{2} + \frac{\pi}{4}\right) = \frac{\pi}{4}$  (triangle  $\triangle OAC$ )

BY THE SINE RULE ON  $\triangle AOD$

$$\frac{|OA|}{\sin \frac{\pi}{6}} = \frac{|OD|}{\sin \frac{\pi}{12}} \Rightarrow \frac{2}{\sin \frac{\pi}{6}} = \frac{12}{\sin \frac{\pi}{12}}$$

$$\Rightarrow 2 = \frac{12 \sin \frac{\pi}{6}}{\sin \frac{\pi}{12}}$$

$$\Rightarrow \underline{2 = 6\sqrt{6} + 6\sqrt{2}}$$

( $\approx 23.18$ )

b) AREA OF THE SECTOR = " $\frac{1}{2}r^2\theta$ "

$$= \frac{1}{2}2^2 \times \frac{\pi}{4}$$

$$= \frac{1}{2}(6\sqrt{6} + 6\sqrt{2})^2 \times \frac{\pi}{4}$$

$$= \frac{\pi}{8} [6(\sqrt{6} + \sqrt{2})]^2$$

$$= \frac{\pi}{8} \times 6^2 (\sqrt{6} + \sqrt{2})^2$$

$$= \frac{\pi}{8} \times 36 \times (8 + 4\sqrt{3})$$

$$= \frac{\pi}{8} \times 36 \times 4 \times (2 + \sqrt{3})$$

$$= 18\pi(2 + \sqrt{3}) \approx 211$$

c) NOW WORKING AT  $\triangle AOC$

$$\frac{|OC|}{|OA|} = \cos \frac{\pi}{4} \Rightarrow \frac{y}{2} = \cos \frac{\pi}{4}$$

$$\Rightarrow y = 2 \times \frac{\sqrt{2}}{2}$$

$$\Rightarrow y = (6\sqrt{6} + 6\sqrt{2}) \times \frac{\sqrt{2}}{2}$$

$$\Rightarrow y = (3\sqrt{6} + 3\sqrt{2}) \times \sqrt{2}$$

$$\Rightarrow y = 3\sqrt{12} + 6$$

$$\Rightarrow y = 6 + 6\sqrt{3}$$

1YGB - MP2 PAPER 2 - QUESTION 4FINALLY THE AREA OF THE TRIANGLE  $\triangle OAC$ 

$$\begin{aligned}
 \text{AREA} &= \frac{1}{2} |OA| |OC| \sin \frac{\pi}{4} = \frac{1}{2} xy \times \frac{\sqrt{2}}{2} = \frac{1}{4} \sqrt{2} xy \\
 &= \frac{1}{4} \sqrt{2} (6\sqrt{6} + 6\sqrt{2}) (6 + 6\sqrt{3}) \\
 &= \frac{1}{4} \sqrt{2} \times 6 (\sqrt{6} + \sqrt{2}) \times 6 (1 + \sqrt{3}) = 9\sqrt{2} (\sqrt{6} + \sqrt{2}) (1 + \sqrt{3}) \\
 &= 9\sqrt{2} (2\sqrt{6} + 4\sqrt{2}) = 9\sqrt{2} \times 2 (\sqrt{6} + 2\sqrt{2}) \\
 &= 18(\sqrt{12} + 4) = 18(2\sqrt{3} + 4) = 36(\sqrt{3} + 2)
 \end{aligned}$$

THE SHADDED AREA IS GIVEN BY

$$\begin{aligned}
 &\text{AREA OF SECTOR} - \text{AREA OF TRIANGLE} \\
 &= 18\pi(2 + \sqrt{3}) - 36(2 + \sqrt{3}) \\
 &= 18(2 + \sqrt{3}) [\pi - 2] \\
 &= \underline{18(2 + \sqrt{3})(\pi - 2)}
 \end{aligned}$$

## LYGB - MP2 PAPER 2 - QUESTION 5

LOCATE THE CO.ORDINATES OF THE MINIMUM BY DIFFERENTIATION

$$f(x) = e^{nx} + ke^{-nx}$$

$$f'(x) = ne^{nx} - nke^{-nx}$$

so we  $f'(x) = 0$

$$\Rightarrow ne^{nx} - nke^{-nx} = 0$$

$$\Rightarrow e^{nx} - ke^{-nx} = 0 \quad n \neq 0$$

$$\Rightarrow e^{nx} = ke^{-nx}$$

$$\Rightarrow e^{nx} = \frac{k}{e^{nx}}$$

$$\Rightarrow (e^{nx})^2 = k$$

$$\Rightarrow e^{nx} = \pm\sqrt{k} \quad e^{nx} > 0$$

NEXT WE CAN FIND THE  $y$  CO.ORDINATE - WE DON'T REQUIRE  $x$

$$\Rightarrow y = e^{nx} + ke^{-nx}$$

$$\Rightarrow y = e^{nx} + \frac{k}{e^{nx}}$$

$$\Rightarrow y = \sqrt{k} + \frac{k}{\sqrt{k}}$$

$$\Rightarrow y = \sqrt{k} + \sqrt{k}$$

$$\Rightarrow y = 2\sqrt{k}$$

$\therefore$  THE RANGE IS  $f(x) \geq 2\sqrt{k}$

# LYGB - MP2 PAPER 2 - QUESTION 6

## a) COLLECTING ALL THE INFORMATION

$$\frac{dV}{dt} = -kV^2$$

↑ RATE  
↑↑↑ DEPRECIATING  
↑↑↑ VALUE SQUARED PROPORTIONAL

V = value, in thousands  
t = time, in years  
-----  
t = 0, V = 12

## SOLVING BY SEPARATING VARIABLES

$$\begin{aligned} \Rightarrow dV &= -kV^2 dt \\ \Rightarrow -\frac{1}{V^2} dV &= k dt \\ \Rightarrow \int -\frac{1}{V^2} dV &= \int k dt \\ \Rightarrow \frac{1}{V} &= kt + C \end{aligned}$$

## APPLY CONDITION t=0, V=12

$$\begin{aligned} \Rightarrow \frac{1}{12} &= C \\ \Rightarrow \frac{1}{V} &= kt + \frac{1}{12} \\ \Rightarrow \dot{V} &= \frac{1}{kt + \frac{1}{12}} \\ \Rightarrow V &= \frac{12}{12kt + 1} \end{aligned}$$

MULTIPLY TOP & BOTTOM OF THE FRACTION IN THE R.H.S BY 12

$$\Rightarrow V = \frac{12}{12kt + 1}$$

AS REQUIRED

1YGB - MP2 PAPER 2 - QUESTION 6

b) USING THE FINAL CONDITION

$$t=0 \quad V=8 \quad \leftarrow \text{£12000} - \text{£4000}$$

$$\Rightarrow 8 = \frac{12}{2a+1}$$

$$\Rightarrow 16a+8=12$$

$$\Rightarrow 16a=4$$

$$\Rightarrow a = \frac{1}{4}$$

REWRITING THE FORMULA

$$\Rightarrow V = \frac{12}{\frac{1}{4}t+1}$$

$$\Rightarrow V = \frac{12}{\frac{1}{4} \times 12 + 1} \quad (\text{'1' FURTHER PERIOD ...})$$

$$\Rightarrow V = 3$$

$$\therefore \text{£} \underline{3000}$$

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# LYGB - MP2 PAPER 2 - QUESTION 7

a)

FILL IN THE TABLE

$x$	$\frac{\pi}{6}$	$\frac{5\pi}{24}$	$\frac{\pi}{4}$	$\frac{7\pi}{24}$	$\frac{\pi}{3}$
$y$	3	4.1120	5.8284	8.6784	13.9282

BY THE TRAPEZIUM RULE

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(1+\sin x)^2}{\cos^2 x} dx \approx \frac{\text{"THICKNESS"}}{2} \left[ \text{FIRST} + \text{LAST} + 2 \times (\text{SUM OF REST}) \right]$$
$$\approx \frac{\frac{\pi}{24}}{2} \left[ 3 + 13.9282 + 2(4.1120 + 5.8284 + 8.6784) \right]$$
$$\approx \underline{3.545}$$

b)

PROCEED BY DIRECT INTEGRATION

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(1+\sin x)^2}{\cos^2 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1+2\sin x + \sin^2 x}{\cos^2 x} dx$$
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + \frac{2\sin x}{\cos x} \cdot \frac{1}{\cos x} + \tan^2 x dx$$
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + 2\tan x \sec x + (\sec^2 x - 1) dx$$
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2 x + 2\tan x \sec x - 1 dx$$

IYGB - MP2 PAPER 2 - QUESTION 7

NOW WE NOTE THAT

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad \& \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$

HENCE WE FINALLY HAVE

$$\begin{aligned} \dots &= \left[ 2 \tan x + 2 \sec x - x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\ &= \left( 2 \tan \frac{\pi}{3} + 2 \sec \frac{\pi}{3} - \frac{\pi}{3} \right) - \left( 2 \tan \frac{\pi}{6} + 2 \sec \frac{\pi}{6} - \frac{\pi}{6} \right) \\ &= \left( 2\sqrt{3} + 4 - \frac{\pi}{3} \right) - \left( \frac{2}{\sqrt{3}} + \frac{4}{\sqrt{3}} - \frac{\pi}{6} \right) \\ &= \left( 2\sqrt{3} + 4 - \frac{\pi}{3} \right) - \left( \frac{6}{\sqrt{3}} - \frac{\pi}{6} \right) \\ &= \cancel{2\sqrt{3}} + 4 - \frac{\pi}{3} - \left( \cancel{2\sqrt{3}} - \frac{\pi}{6} \right) \\ &= 4 - \frac{\pi}{6} \end{aligned}$$

1YGB - MP2 PAPER 2 - QUESTION 8

a) START BY REARRANGING THE EQUATION FOR x - THEN DIFFERENTIATE

$$\Rightarrow y = \frac{x}{y + \ln y}$$

$$\Rightarrow y^2 + y \ln y = x$$

$$\Rightarrow x = y^2 + y \ln y$$

$$\Rightarrow \frac{dx}{dx} = 2y + 1 \times \ln y + y \times \frac{1}{y}$$

$$\Rightarrow \frac{dx}{dy} = 2y + \ln y + 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y + \ln y + 1}$$

SOLVING  $\frac{dy}{dx} = 2$  YIELD

$$\Rightarrow 2 = \frac{1}{2y + \ln y + 1}$$

$$\Rightarrow 4y + 2 \ln y + 2 = 1$$

$$\Rightarrow 4y + 2 \ln y + 1 = 0$$

$$\Rightarrow 2 \ln y = -1 - 4y$$

$$\Rightarrow \ln y = -\frac{1}{2} - 2y$$

$$\Rightarrow \ln y = -\frac{1}{2}(4y + 1)$$

$$\Rightarrow y = e^{-\frac{1}{2}(4y + 1)}$$

AS REQUIRED

b) USING THE ITERATION FORMULA  $y = e^{-\frac{1}{2}(4y + 1)}$

STARTING WITH  $y_1 = 0.3$

$$y_2 = 0.33287 \dots$$

$$y_3 = 0.311691 \dots$$

$$y_4 = 0.325178 \dots$$

# 1YGB - MP2 PAPER 2 - QUESTION 8

THE CONVERGENCE IS BY OSCILLATION BUT VERY SLOW

$$y_5 = 0.316524\dots$$

$$y_6 = 0.32205\dots$$

$$y_7 = 0.31851\dots$$

$$y_8 = 0.32077\dots$$

$$y_9 = 0.31932\dots$$

$$y_{10} = 0.32025\dots$$

$$y_{11} = 0.31965\dots$$

$$y_{12} = 0.32003\dots$$

$$y_{13} = 0.31979\dots$$

$$y_{14} = 0.31995\dots$$

$$y_{15} = 0.31985\dots$$

$$\therefore \underline{y = 0.320} \text{ (CORRECT TO 3 d.p.)}$$

USING  $y = 0.3199$  IN  $\alpha = y^2 + y \ln y$  WE OBTAIN  $\alpha = -0.262$

$$\therefore \underline{P(-0.262, 0.320)}$$

# 1YGB - MP2 PART 2 R - QUESTION 9

a) 
$$f(x) \equiv \frac{16x^2 + 3x - 2}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{3x-2}$$

$$16x^2 + 3x - 2 \equiv A(3x-2) + Bx(3x-2) + Cx^2$$

• IF  $x=0$

$$-2 = -2A$$

$$A = 1$$

• IF  $x = \frac{2}{3}$

$$\frac{64}{9} + 2 - 2 = C \times \frac{4}{9}$$

$$C = 16$$

• IF  $x=1$

$$17 = A + B + C$$

$$17 = 1 + B + 16$$

$$B = 0$$

b) 
$$\frac{1}{3x-2} = -\frac{1}{2-3x} = -(2-3x)^{-1} = -(2)^{-1} \left[1 - \frac{3}{2}x\right]^{-1}$$

$$= -\frac{1}{2} \left(1 - \frac{3}{2}x\right)^{-1}$$

$$= -\frac{1}{2} \left[ 1 + \frac{-1}{1} \left(-\frac{3}{2}x\right)^1 + \frac{-1(-2)}{1 \times 2} \left(-\frac{3}{2}x\right)^2 + \frac{(-1)(-2)(-3)}{1 \times 2 \times 3} \left(-\frac{3}{2}x\right)^3 + \dots \right]$$

$$= -\frac{1}{2} \left[ 1 + \frac{3}{2}x + \frac{9}{4}x^2 + \frac{27}{8}x^3 + \dots \right]$$

$$= -\frac{1}{2} - \frac{3}{4}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 - \dots$$

c) Method A (only up to  $x^3$  is directly available)

$$\frac{16x^2 + 3x - 2}{3x-2} = (-2 + 3x + 16x^2) \left[ -\frac{1}{2} - \frac{3}{4}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 + \dots \right]$$

$$= 1 + \cancel{\frac{3}{2}x} + \cancel{\frac{9}{4}x^2} + \cancel{\frac{27}{8}x^3} + \dots$$

$$- \cancel{\frac{3}{2}x} - \cancel{\frac{9}{4}x^2} - \cancel{\frac{27}{8}x^3} + \dots$$

$$- 8x^2 - 12x^3 + \dots$$

$$= \underline{1 - 8x^2 - 12x^3 + \dots}$$

1YGB - MP2 PAPER 2 - QUESTION 9

METHOD B (USING PREVIOUS PARTS)

$$\frac{16x^2 + 3x - 2}{x^2(3x-2)} = \frac{1}{x^2} + \frac{16}{3x-2}$$

$$\frac{1}{x^2} \left( \frac{16x^2 + 3x - 2}{3x-2} \right) = \frac{1}{x^2} + 16 \left( \frac{1}{3x-2} \right)$$

$$\frac{16x^2 + 3x - 2}{3x-2} = 1 + 16x^2 \left( \frac{1}{3x-2} \right)$$

$$\frac{16x^2 + 3x - 2}{3x-2} = 1 + 16x^2 \left[ -\frac{1}{2} - \frac{3}{4}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 + O(x^4) \right]$$

$$\frac{16x^2 + 3x - 2}{3x-2} = 1 - 8x^2 - 12x^3 - 18x^4 - 27x^5 + O(x^6)$$

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# IYGB - MP2 PAPER R - QUESTION 10

a) STARTING FROM THE L.H.S

$$\begin{aligned} \text{LHS} &= \sin 3\theta \\ &= \sin(2\theta + \theta) \\ &= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \sin(A+B) \equiv \sin A \cos B + \cos A \sin B \\ &= (2\sin \theta \cos \theta) \cos \theta + (1 - 2\sin^2 \theta) \sin \theta \\ &= 2\sin \theta \cos^2 \theta + \sin \theta + 2\sin^3 \theta \\ &= 2\sin \theta (1 - \sin^2 \theta) + \sin \theta + 2\sin^3 \theta \\ &= 2\sin \theta - 2\sin^3 \theta + \sin \theta + 2\sin^3 \theta \\ &= 3\sin \theta - 4\sin^3 \theta \\ &= \underline{\text{RHS}} \end{aligned}$$

// AS REQUIRED

b) DIFFERENTIATING THE IDENTITY W.R.T  $\theta$

$$\begin{aligned} \frac{d}{d\theta} [\sin 3\theta] &= \frac{d}{d\theta} [3\sin \theta - 4\sin^3 \theta] \\ 3\cos 3\theta &= 3\cos \theta - 12\sin^2 \theta \times \cos \theta \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \div 3 \\ \cos 3\theta &= \cos \theta - 4\cos \theta \sin^2 \theta \\ \cos 3\theta &= \cos \theta - 4\cos \theta (1 - \cos^2 \theta) \\ \cos 3\theta &= \cos \theta - 4\cos \theta + 4\cos^3 \theta \\ \cos 3\theta &= \underline{4\cos^3 \theta - 3\cos \theta} \end{aligned}$$

// AS REQUIRED

c) PROCEED AS FOLLOWS

$$\begin{aligned} \tan 3\theta &= \frac{\sin 3\theta}{\cos 3\theta} = \frac{3\sin \theta - 4\sin^3 \theta}{4\cos^3 \theta - 3\cos \theta} = \frac{\frac{3\sin \theta}{\cos^3 \theta} - \frac{4\sin^3 \theta}{\cos^3 \theta}}{\frac{4\cos^3 \theta}{\cos^3 \theta} - \frac{3\cos \theta}{\cos^3 \theta}} \\ &= \frac{\frac{3\sin \theta}{\cos \theta} \times \frac{1}{\cos^2 \theta} - 4\tan^3 \theta}{4 - \frac{3}{\cos^2 \theta}} = \frac{3\tan \theta \sec^2 \theta - 4\tan^3 \theta}{4 - 3\sec^2 \theta} \end{aligned}$$

// AS REQUIRED

14GB - MP2 PAPER R - QUESTION 10

d) USING THE IDENTITY  $1 + \tan^2\theta \equiv \sec^2\theta$  WE HAVE

$$\tan 3\theta = \frac{3 \tan\theta \sec^2\theta - 4 \tan^3\theta}{4 - 3 \sec^2\theta} = \frac{3 \tan\theta (1 + \tan^2\theta) - 4 \tan^3\theta}{4 - 3(1 + \tan^2\theta)}$$

$$= \frac{3 \tan\theta + 3 \tan^3\theta - 4 \tan^3\theta}{4 - 3 - 3 \tan^2\theta} = \frac{3 \tan\theta - \tan^3\theta}{1 - 3 \tan^2\theta}$$

~~AS REQUIRED~~

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## IYGB - MP2 PAPER 2 - QUESTION 11

a) OBTAIN THE GRADIENT FUNCTION IN PARAMETRIC

$$\bullet \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\sin^2 t \cos t}{3\cos^2(-\sin t)} = - \frac{3\sin^2 t \cos t}{3\sin t \cos^2 t} = - \frac{\sin t}{\cos t}$$

$$\bullet \left. \frac{dy}{dx} \right|_{t=\theta} = - \frac{\sin \theta}{\cos \theta}$$

EQUATION OF NORMAL AT  $(\cos^3 \theta, \sin^3 \theta)$  WITH GRADIENT  $+$   $\frac{\cos \theta}{\sin \theta}$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - \sin^3 \theta = \frac{\cos \theta}{\sin \theta} (x - \cos^3 \theta)$$

$$\Rightarrow y \sin \theta - \sin^4 \theta = x \cos \theta - \cos^4 \theta$$

$$\Rightarrow \cos^4 \theta - \sin^4 \theta = x \cos \theta - y \sin \theta$$

$$\Rightarrow \underbrace{(\cos^2 \theta - \sin^2 \theta)}_{\cos 2\theta} \underbrace{(\cos^2 \theta + \sin^2 \theta)}_1 = x \cos \theta - y \sin \theta$$

$$\Rightarrow \underline{\underline{x \cos \theta - y \sin \theta = \cos 2\theta}}$$

As required

b) When  $x=0$

$$-y \sin \theta = \cos 2\theta$$

$$y = - \frac{\cos 2\theta}{\sin \theta}$$

When  $y=0$

$$x \cos \theta = \cos 2\theta$$

$$x = \frac{\cos 2\theta}{\cos \theta}$$

AREA IS GIVEN BY

$$\frac{1}{2} \left| - \frac{\cos 2\theta}{\sin \theta} \times \frac{\cos 2\theta}{\cos \theta} \right| = \frac{\cos 2\theta \cos 2\theta}{2 \sin \theta \cos \theta} = \frac{\cos 2\theta \cos 2\theta}{\sin 2\theta}$$

$$= \underline{\underline{\cos 2\theta \cos 2\theta}}$$