IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper Q

Difficulty Rating: 3.875/1.3176

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 13 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

It is asserted that

$$|2x+1| \le 5 \Rightarrow |x| \le 2 .$$

Disprove this assertion by a **counter-example**. **(3)**

Question 2

Relative to a fixed origin O, the point A has coordinates $(2,5,4)$.

The points B, C and D are such so that

$$\overrightarrow{BA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} , \quad \overrightarrow{BC} = 7\mathbf{i} + \mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{DC} = 4\mathbf{i} + 2\mathbf{k} .$$

Determine the distance of D from the origin. **(6)**

Question 3

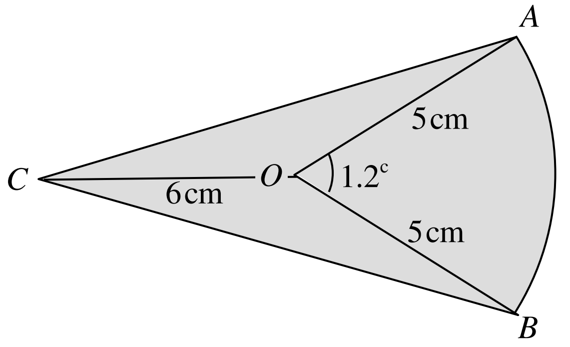
An area of neglected lawn is treated with weed killer. Before the treatment started, the area covered by the weed was 75 m^2 and two days later it has reduced to 33.7 m^2.

Let the area of the lawn covered with weed be $A \text{ m}^2$, t days after it was treated.

The rate at which the area covered by the weed is decreasing, is proportional to the area still covered by the weed.

By forming and solving a suitable differential equation, express A in terms of t. **(7)**

Question 4



The figure above shows a template design CAB.

The curve AB is the arc of a circular sector OAB, subtending an angle of 1.2 radians at its centre O. The radius of the sector is 5 cm. The straight lines CA and CB are of equal length. The length of the straight line OC is 6 cm.

Find, to three significant figures where appropriate, …

a) … the area of the circular sector OAB. **(2)**

b) … the size of the angle COB, in radians. **(2)**

c) … the total area of the template design. **(3)**

Question 5

It is given that the value of

$$\int_0^{\frac{1}{3}\pi} \left(k\cos^2 x - \sec^2 x\right)\sin x \; dx,$$

is 2, where k is a non zero constant.

Determine the value of k. **(7)**

Question 6

$$f(x) = \sqrt{1-x}\,,\ -1 < x < 1\,.$$

a) Expand $f(x)$ up and including the term in x^3. **(3)**

b) Show clearly that

$$7\sqrt{1-\frac{1}{49}} = 4\sqrt{3}\,.$$ **(1)**

c) By using the **first two** terms of the expansion obtained in part **(a)** and the result obtained in part **(b)**, show further that

$$\sqrt{3} \approx \frac{97}{56}\,.$$ **(4)**

Question 7

It is required to find the approximate coordinates of the points of intersection between the graphs of

$$y_1 = 1 - x^2,\ x \in \mathbb{R} \qquad \text{and} \qquad y_2 = \ln(x+1),\ x \in \mathbb{R},\ x > -1\,.$$

a) Show that the two graphs intersect at a single point P, explaining further why the x coordinate of P lies between 0 and 1. **(2)**

b) Use the Newton Raphson method once, starting $x = 0.7$, to calculate the x coordinate of P, giving the answer correct to 3 decimal places. **(6)**

c) By considering two suitable transformations, determine correct to 2 decimal places the coordinates of the points of intersection between the graph of

$$y_3 = 2\left[1-(2x+1)^2\right],\ x \in \mathbb{R} \qquad \text{and} \qquad y_4 = 2\ln(2x+1),\ x \in \mathbb{R},\ x > -\tfrac{1}{2}\,.$$ **(2)**

Question 8

It is given that for $\theta \in \mathbb{R}$, $\varphi \in \mathbb{R}$

$$3\tan\theta = 4\tan\varphi.$$

Show that the above relationship implies that

$$\tan(\theta - \varphi) = \frac{\sin 2\theta}{7 + \cos 2\theta}. \qquad (7)$$

Question 9

The functions f and g are defined by

$$f(x) = x^2 - 4, \ x \in \mathbb{R}, \ x > 8$$

$$g(x) = 2x - 2, \ x \in \mathbb{R}, \ x > 3.$$

a) State the range of $f(x)$ and the range of $g(x)$. **(2)**

b) Find a simplified expression for $fg(x)$. **(2)**

c) Determine the domain and range of $fg(x)$. **(2)**

Question 10

A bubble is formed and its volume is increasing at the constant rate of $20 \text{ cm}^3\text{s}^{-1}$.

The shape of the bubble remains spherical at all times.

Find the rate at which the radius of the bubble is increasing …

a) … when the radius of the bubble reaches 5 cm. **(3)**

b) ... when the volume of the bubble reaches 300 cm^3. **(4)**

c) … ten seconds after the bubble was first formed. **(4)**

$\left[\text{volume of a sphere of radius } r \text{ is given by } \frac{4}{3}\pi r^3\right]$

Question 11

The curve C has equation given by

$$y = \frac{e^x}{\sin x}, \quad 0 < x < \pi .$$

a) Show clearly that

$$\frac{dy}{dx} = y(1 - \cot x) .$$ **(4)**

b) Show further that

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}(1 - \cot x) + y\operatorname{cosec}^2 x .$$ **(2)**

c) Use the above results to find the exact coordinates of the turning point of C, and determine its nature. **(4)**

Question 12

A curve is given parametrically by the equations

$$x = 3\cos t, \quad y = 4\sin t, \quad 0 \le t \le 2\pi .$$

a) Show that the equation of the tangent to the curve at the point where $t = \theta$ is

$$3y\sin\theta + 4x\cos\theta = 12 .$$

(5)

The tangent to the curve at the point where $t = \theta$ meets the y axis at the point $P(0,8)$ and the x axis at the point Q.

b) Find the exact area of the triangle POQ, where O is the origin. **(7)**

Question 13

It is given that

$$\sum_{r=1}^{20}\left[f(r) - 10\right] = 200 \quad \text{and} \quad \sum_{r=1}^{20}\left[f(r) - 10\right]^2 = 2800 .$$

Find the value of

$$\sum_{r=1}^{20}\left[f(r)\right]^2 .$$

(6)
